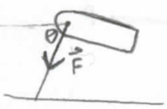
Chapter 5

Equilibrium of a Rigid Body

Support:
Something that prevents translation or rotation in a given direction

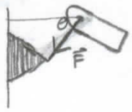
Two-Dimensional Supports

① **Cable:** $\boxed{\text{Unkowns}}$



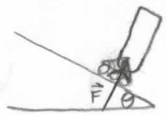
1, tension force in direction of the cable

② **Weightless Link:** $\boxed{\text{Unknowns}}$



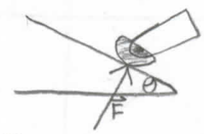
1, force along the axis of the link

③ **Roller:** $\boxed{\text{Unknowns}}$



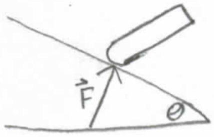
1, Force that acts perpendicular to the surface

④ **Rocker:** $\boxed{\text{Unknowns}}$



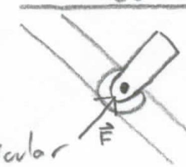
1, Force that acts perpendicular to the surface

⑤ **Smooth contact surface:** $\boxed{\text{Unknowns}}$



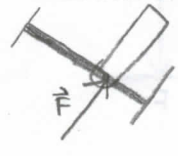
1, force perpendicular to the contact surface

⑥ **Roller or Pin in Confined Smooth slot:** $\boxed{\text{Unknowns}}$



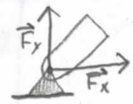
1, Force ⊥ to the slot

⑦ **Pin Connected to Collar on Rod:** $\boxed{\text{Unknowns}}$



1, $\vec{F}$ ⊥ to the rod

⑧ **Pin or hinge:** $\boxed{\text{Unknowns}}$



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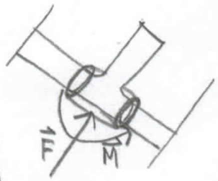
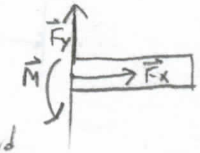
⑨ **Collar:** $\boxed{\text{Unknowns}}$



2, Force and moment

⑩ **Fixed Support:** $\boxed{\text{Unknowns}}$



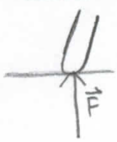
3, Force components and moment

Three-Dimensional Supports

① **Cable:** $\boxed{\text{Unknowns}}$

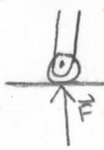
1, Force in direction of the cable

② **Smooth Surface Support:** $\boxed{\text{Unknowns}}$



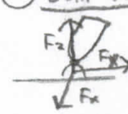
1 $\vec{F}$ ⊥ to contact surface

③ **Roller:** $\boxed{\text{Unknowns}}$



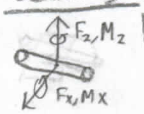
1, $\vec{F}$ ⊥ to contact surface

④ **Ball and Socket:** $\boxed{\text{Unknowns}}$



3, directions that it cannot translate

⑤ **Circular Journal Bearing:** $\boxed{\text{Unknowns}}$



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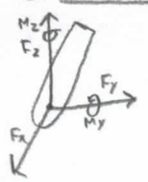
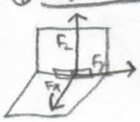
⑧ **Single smooth pin:** $\boxed{\text{Unknowns}}$



5

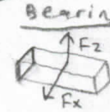
⑨ **Single Hinge:** $\boxed{\text{Unknowns}}$



F_z, F_x, F_y, M_z, M_x

5

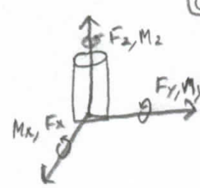
⑥ **Square Journal Bearing:** $\boxed{\text{Unknowns}}$



F_z, F_x
M_z, M_y, M_x

5

⑩ **Fixed Support:** $\boxed{\text{Unknowns}}$



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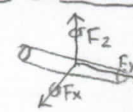
⑦ **Thrust Bearing:**



F_z, F_x, F_y
M_z, M_x

5

Equations of Equilibrium

Vector equations:

$$\sum F = 0$$
$$\sum M_o = 0$$

Scalar Equations:

$$\sum F_x = 0 \quad \sum M_x = 0$$
$$\sum F_y = 0 \quad \sum M_y = 0$$
$$\sum F_z = 0 \quad \sum M_z = 0$$

Linearly Elastic Spring:

$\vec{F} = ks$

where s is distance from equilibrium position

$\Delta x = V_0 t + \frac{1}{2} a t^2$

$V_f = V_0 + at$

$V_f^2 = V_0^2 + 2a\Delta x$

$W_f^2 = W_0^2 + 2\alpha \Delta\theta$

Particle Equilibrium

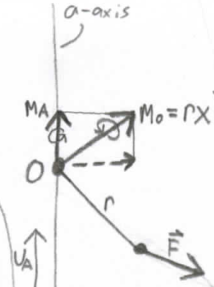
$\Sigma F = 0 \quad \Sigma F = ma$

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma F_z = 0$

Moment of force about a specific axis:



$M_A = [U_{Ax}\hat{\imath} + U_{Ay}\hat{\jmath} + U_{Az}\hat{k}] \cdot \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$

*dot

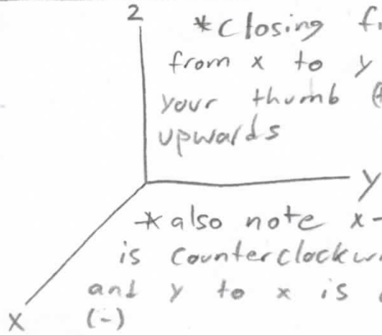
$= \vec{U_A} \cdot (r \times F) = \begin{vmatrix} U_{Ax} & U_{Ay} & U_{Az} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$

$= U_{Ax}(r_y F_z - r_z F_y)\hat{\imath}$

$- U_{Ay}(r_x F_z - r_z F_x)\hat{\jmath}$

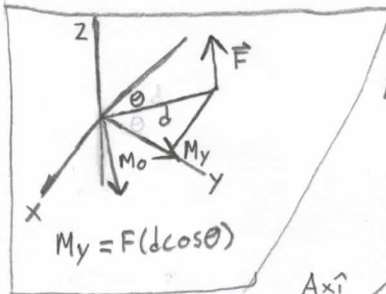
$+ U_{Az}(r_x F_y - r_y F_x)\hat{k}$

Right handed coordinate System

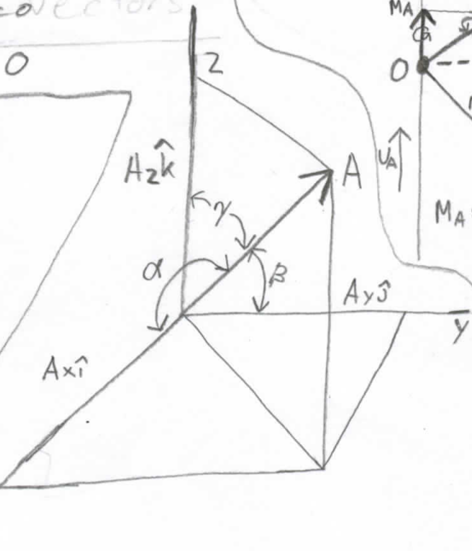


*Closing fingers from x to y points your thumb (+) positive upwards

*also note x-y is counterclockwise (+) and y to x is clockwise (−)



$M_y = F(d\cos\theta)$



$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} = [A_x, A_y, A_z]$

$\|\vec{A}\| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$

Unit Vector

• Dimensionless with magnitude equal to 1

$U_A = \frac{\vec{A}}{\|\vec{A}\|} = \frac{A_x}{\|\vec{A}\|}\hat{\imath} + \frac{A_y}{\|\vec{A}\|}\hat{\jmath} + \frac{A_z}{\|\vec{A}\|}\hat{k}$

$U_A = \cos\alpha\,\hat{\imath} + \cos\beta\,\hat{\jmath} + \cos\gamma\,\hat{k}$

Coordinate Direction Angles $0 \le \theta \le 180°$

$\cos\alpha = \frac{A_x}{\|\vec{A}\|} \quad \cos\beta = \frac{A_y}{\|\vec{A}\|} \quad \cos\gamma = \frac{A_z}{\|\vec{A}\|}$

Identity: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

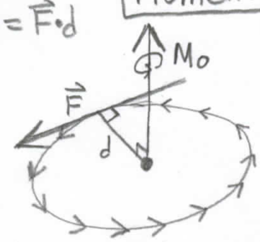
Dot Product $0 \le \theta \le 180°$

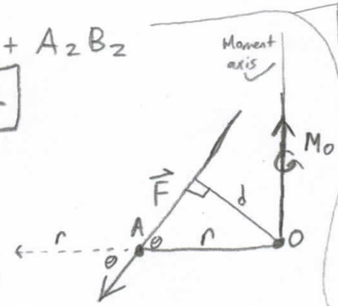
$\vec{A} \cdot \vec{B} = \|\vec{A}\|\|\vec{B}\|\cos\theta$

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Moment

$\|\vec{M_0}\| = \vec{F} \cdot d$





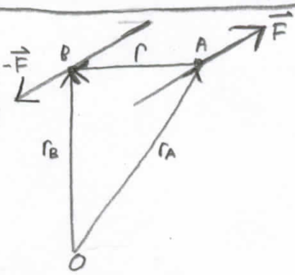
$M_0 = rF\sin\theta = Fd$

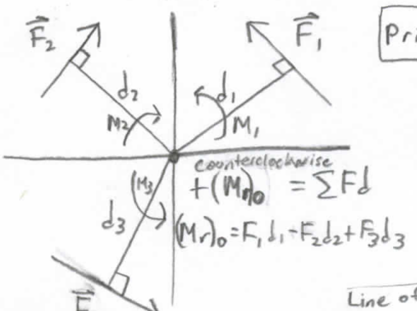
Moment of a Couple

$M = (r_B \times (-\vec{F})) + (r_A \times \vec{F})$

$\therefore M = r \times F$

"If $\vec{F}$ and $(-\vec{F})$ are equal in magnitude and anti-parallel"



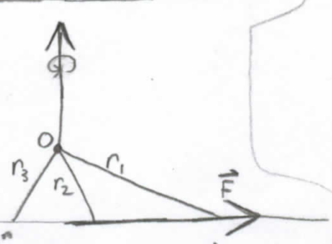


counterclockwise

$+(M_R)_0 = \Sigma Fd$

$(M_R)_0 = F_1 d_1 - F_2 d_2 + F_3 d_3$

Principle of transmissibility



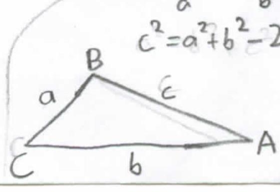
$M_0 = r_1 \times \vec{F} = r_2 \times \vec{F} = r_3 \times \vec{F}$

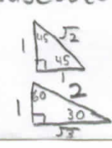
$M_0 = r \times F = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$

$M_0 = (r_y F_z - r_z F_y)\hat{\imath} - (r_x F_z - r_z F_x)\hat{\jmath} + (r_x F_y - r_y F_x)\hat{k}$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

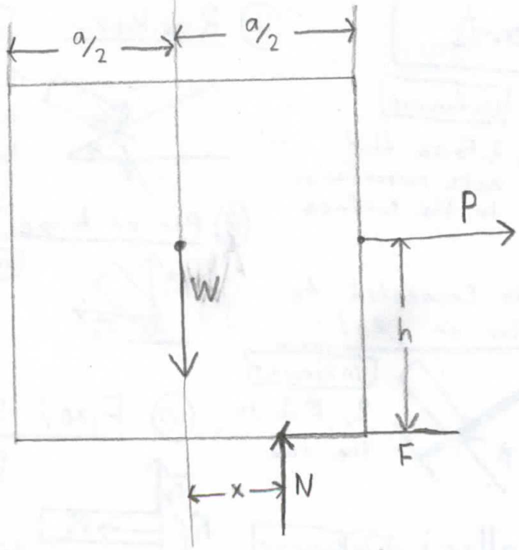
$c^2 = a^2 + b^2 - 2ab\cos(C)$





Chapter 8

Friction



Equilibrium Conditions

$$F = P$$
$$N = W$$
$$Wx = Ph$$

Impending Motion